Bell Ringer: Centripetal Acceleration – ID: 13608

Time required
15 minutes

Topic: Circular Motion

- *Explore the motion of an object traveling in a circle.*
- *Derive the equation for the magnitude of centripetal acceleration.*

Activity Overview

In this activity, students will observe a simulation of an object traveling in uniform circular motion. Then, students will derive the equation for the magnitude of centripetal acceleration as a function of tangential velocity and circle radius.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes

Teacher Preparation

Before carrying out this activity, review with students vector addition and resolution and the concept of the tangent to a circle.

- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on page 6 for a preview of the student TI-Nspire document (.tns file). The solution .tns file contains sample responses to the questions posed in the student .tns file.*
- ***To download the student .tns file and solution .tns file, go to education.ti.com/exchange and enter "13608" in the search box.***
- *This activity is related to activity 9746: Uniform Circular Motion. If you wish, you may extend this bell-ringer activity with the longer activity. You can download the files for activity 9746 at education.ti.com/exchange.*

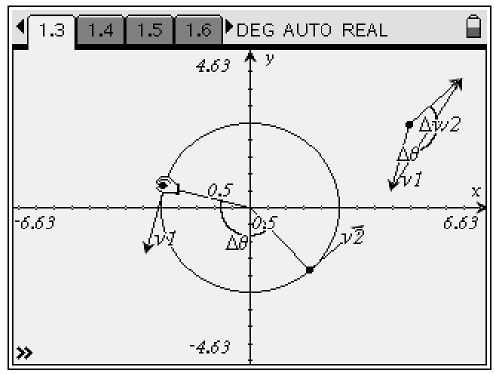
Classroom Management

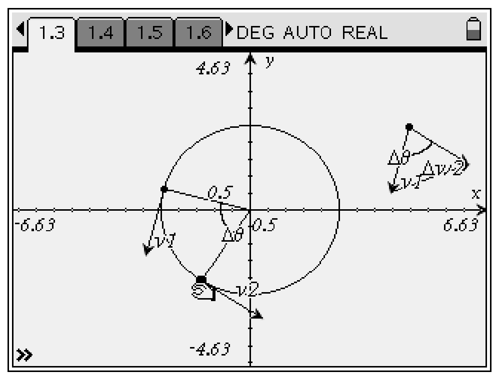
- *This activity is designed to be **teacher-led**, with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *If you wish, you may modify this document for use as a student instruction sheet. You may also wish to use an overhead projector and TI-Nspire computer software to demonstrate the use of the TI-Nspire to students.*
- *If students do not have sufficient time to complete the main questions, they may also be completed as homework.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration during this activity:

- What is uniform circular motion?
- How does the velocity of an object change as it travels in uniform circular motion?
- How do you determine the displacement between two vectors?

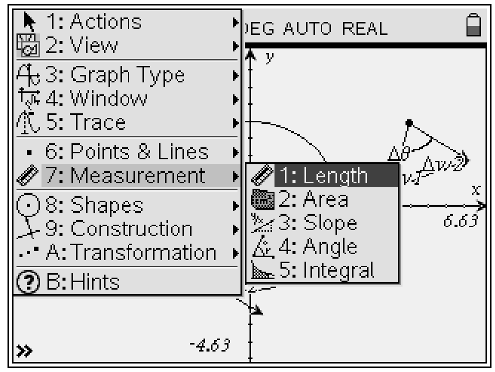
The purpose of this activity is for students to observe the motion of an object traveling in uniform circular motion with respect to the velocity of the object, radius of orbit, and angular position of the object as it moves around the circle. Then, students will use the concept of similar triangles to derive an equation for the centripetal acceleration of the object.





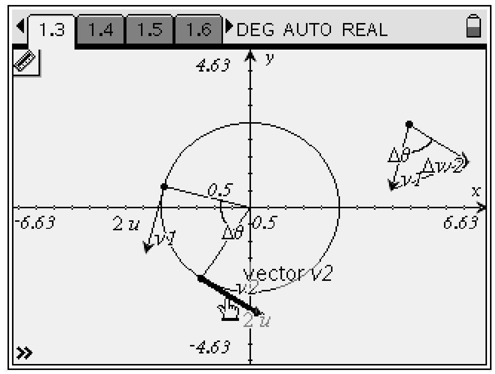
Step 1: Students should open the file **PhysBR_week19_circularmotion.tns** and read the first two pages. Page 1.3 shows the path of an object traveling in uniform circular motion. The vectors *v1* and *v2* represent the tangential velocity of the object at the initial and final times, respectively. The total angular displacement of the object over this time period is given by $\Delta\theta$. To the right of the circular path, the vectors *v1* and *v2* are placed tail to tail. The resultant vector gives the change in velocity, Δv, of the object over the time period. That is, $\Delta v = v2 - v1$. Students can vary the position of the initial and final velocity vectors. (To move a velocity vector, students should use the NavPad to move the cursor to the point along the circle at which the tangent velocity vector originates. The cursor should change to an open hand. Students should press and hold (click) until the cursor changes to a closed hand. They can then use the NavPad to drag the point around the circle. To release the points, students should press (click) again.) Students should observe how the angular displacement and change in velocity are affected by changes in the initial and final vectors.

Step 2: Next, students should answer questions 1–3 on page 1.4.



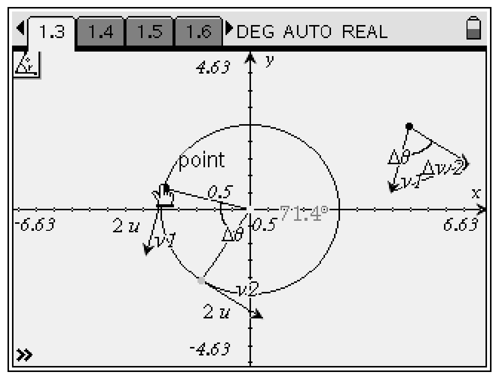
Q1. What can you conclude about the magnitude of the object's velocity based on the fact that the object is traveling in uniform circular motion? Verify this on page 1.3.

A. *The magnitude of the velocity of an object traveling in uniform circular motion is constant. (That is, the speed of the object is constant.) The only change in velocity (due to the centripetal acceleration) is a change in the direction of the velocity. To verify that the magnitude of the velocity is constant using the handheld device, students should return to page 1.3 and use the* ***Length*** *tool (****Menu > Measurement > Length****). They should click once (press) on v1 and v2 to determine the length of each. Students will find that the calculated length for each vector is 2 u.*



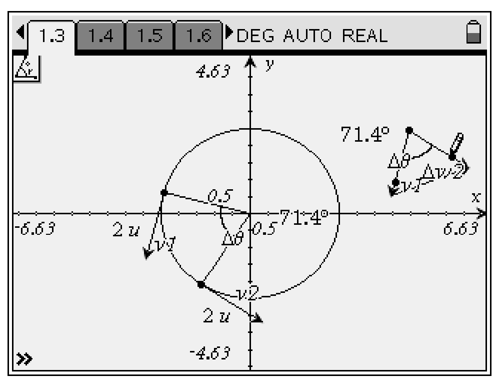
Q2. Verify that the angular displacement of the object traveling around the circle is the same as the angle formed by the tails of the initial and final velocity vectors.

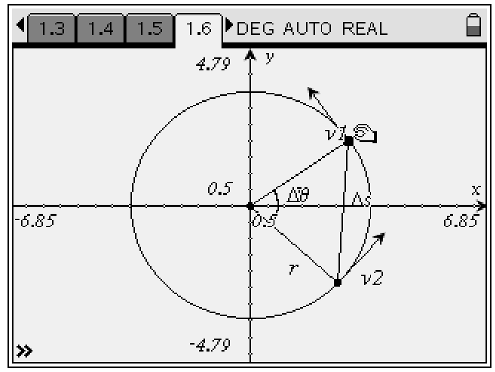
A. *To verify that the angular displacement of the object is the same as the angle formed between the initial and final velocity vectors, students should return to page 1.3 and use the* ***Angle*** *tool (****Menu > Measurement > Angle****). They should click once (press) on each of the three points that define the angle. For example, to determine the angular displacement, students would click on the point at which v1 originates, then they would click on the origin, and then they would click on the point at which v2 originates.*

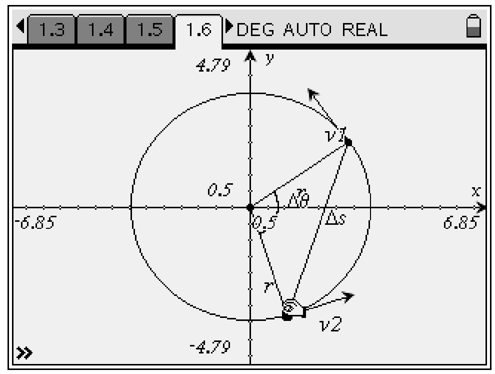


Q3. What kind of a triangle is formed by the tails of the initial and final velocity vectors?

A. *Because the magnitudes of the velocity vectors remain constant, the two sides of the triangle are equal. Thus, the tails of the initial and final velocity vectors form an isosceles triangle.*







Step 3: Next, students should read page 1.5 and then move on to page 1.6. Page 1.6 shows the same circular path and velocity vectors that were represented on page 1.3. In this simulation, the radius of the circular path is given by *r*, and the horizontal displacement between the initial and final positions is given by Δ*s*. Note: for small displacements, this horizontal length is equal to the circular arc over which the object travels. Students can vary the initial and final velocity vectors following the procedure described in Step 1 to observe how the horizontal displacement of the object changes. Then, students should answer questions 4–8.

Q4. What kind of a triangle is formed by the two radii of the circle and the horizontal displacement, Δ*s*?

A. *Because the two sides of the triangle (formed by the two radii of the circle) are the same, this is an isosceles triangle.*

Q5. Note that the triangle formed by the vector tails on page 1.3 is a similar triangle to the triangle formed on page 1.6. Write an expression that relates the base of each triangle to the side of that triangle, and then set these expressions equal to one another.

A. *Two isosceles triangles are similar when the angle between the equivalent sides is the same for both triangles. In this case, the angle Δθ is the same for both triangles. Thus, the expression relating the base to the side of the triangle on page 1.3 is* $\frac{\Delta v}{v}$*; the expression relating the base to the side of the triangle on page 1.6 is* $\frac{\Delta s}{r}$*.*

Equating these expressions gives $\frac{\Delta v}{v} = \frac{\Delta s}{r}$*.*

Q6. Recall that Δ*s* is the distance the object travels in a time Δ*t* traveling at speed *v*. Write an expression for Δ*s* in terms of *v* and Δ*t*.

A. *The average tangential speed, v, is defined as the horizontal displacement over a change in time:* $v = \frac{\Delta s}{\Delta t}$*. Rearranging the variables gives* $\Delta s = v\Delta t$ *.*

Q7. Solve for the centripetal acceleration of the object.

A. *Substituting the expression for Δs into the identity given by the similar triangles gives*

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

Average acceleration is defined as the change in velocity of the object over the change in time, or $a = \frac{\Delta v}{\Delta t}$*. Rearranging the variables in the equation above gives*

$$\frac{\Delta v}{\Delta t} = \frac{v \times v}{r}$$

$$a = \frac{v^2}{r}$$

Q8. An object has a tangential velocity of 2.5 m/s. It is traveling along a circle with a radius of 1.5 m. What is its centripetal acceleration?

A. *The equation for centripetal acceleration is* $a = \frac{v^2}{r}$. Substituting the given values yields

$$a = \frac{v^2}{r}$$

$$a = \frac{(2.5\ m/s)^2}{1.5\ m}$$

$$a = \frac{6.25\ m^2/s^2}{1.5\ m} = 4.2\ m/s^2$$

Suggestions for Extension Activities: Have students derive an equation for the force acting on an object traveling in uniform circular motion. Then, have students calculate the force acting on objects of varying mass, speed, and radius of orbit.

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(Student)TI-Nspire File: *PhysBR_week19_circularmotion.tns*