The Period of a Pendulum – ID: 8735

By Charles W. Eaker

Time required
45 minutes

Topic: Waves

- *Use the equations of motion to find frequency, period, force, energy, velocity, and acceleration of objects in simple harmonic motion.*

Activity Overview

In this activity, students will use a simulated pendulum to collect data on length and period. Then, they will develop mathematical models of the relationship between the length of a pendulum and its period. They will confirm that the relationship is nonlinear and that period is directly proportional to the square root of length.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Notes

Teacher Preparation

Students will probably be familiar with pendulums, but they may not know the quantitative relationship between period and pendulum length. Before beginning this activity, review the concept of the period of a pendulum.

- *The screenshots on pages 2–4 demonstrate expected student results. Refer to the screenshots on pages 5 and 6 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "8735" in the quick search box.***

Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on notebook paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

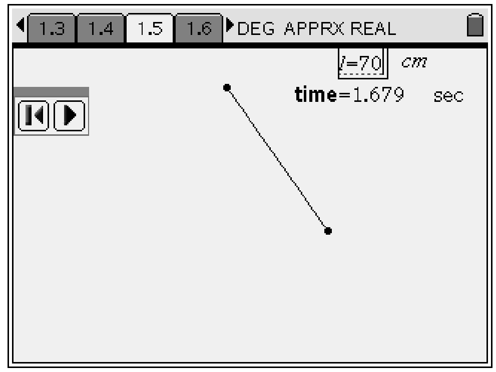
The following questions will guide student exploration in this activity:

- What is the relationship between the length and period of a pendulum?
- How can we create mathematical models based on quantitative data for this relationship?

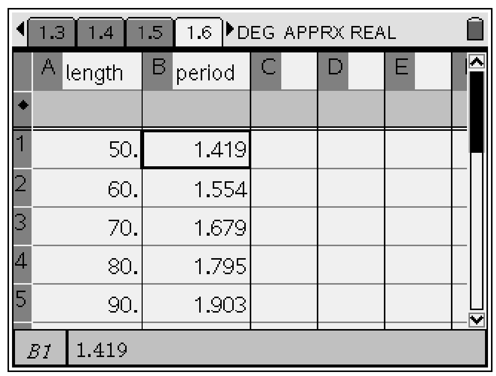
Students will carry out the activity simulating the collection of experimental data. They will use two different approaches to develop an equation relating length and period.

Part 1 – Exploring the length-period relationship

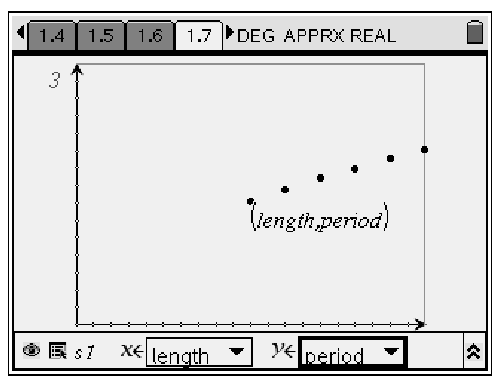
Step 1: Students should open the file **PhysWeek09_pendulum.tns** and read the first four pages. Page 1.5 contains an animation of a pendulum. In the simulation, students will change the length (**l**) of the pendulum, and the program will calculate the period (**time**) for them. Students may use the animation to observe the changing periods of the pendulum. Note: The animated pendulum has a period longer than that displayed (e.g., when **time** = 1.554 sec, the animated pendulum will take longer than 1.554 sec to complete one oscillation).



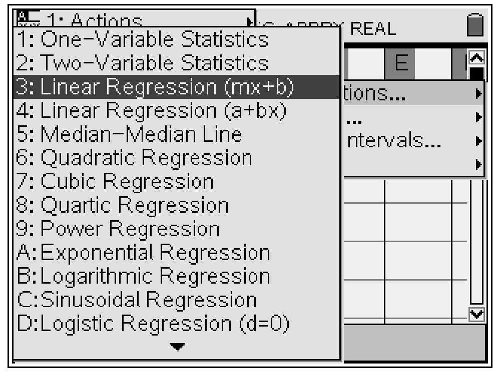
Step 2: Students should record the value of **time** for each of the values of **l** in the *Lists & Spreadsheet* application on page 1.6. Students should observe that increasing the length increases the period. They should enter the time data from page 1.5 into the spreadsheet on page 1.6.



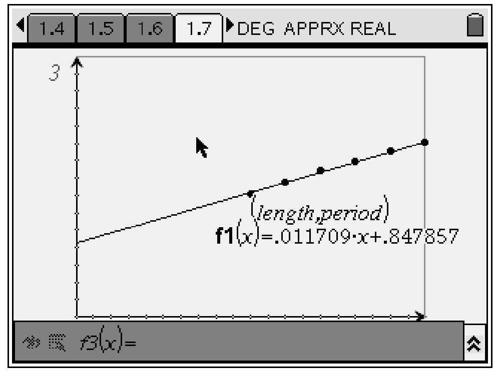
Step 3: Next, the students should create a scatter plot (ctrl menu > **Scatter Plot**) of the data on page 1.7. They should use **length** for the *x*-values and **period** for the *y*-values. Discuss the shape of the graph with the students. Some students will probably assume that this is a linear relationship. Ask them how they could determine whether the relationship is linear. They should reason that a linear regression can provide information about the relationship. You may need to guide them to this conclusion.



Step 4: Next, students should move back to page 1.6 and use a linear regression (**Menu** > **Statistics** > **Stat Calculations** > **Linear Regression (mx + b)**) to find an equation that fits the data points. Students should use **length** for the *X List* and **period** for the *Y List*. They should store the values in Column C of the *Lists & Spreadsheet* application on page 1.6.



Step 5: Next, students should move to page 1.7 and plot their best-fit lines on the scatter plot of **period** vs. **length**. To do this, they should change the plot type to **Function** and then select the function in which the best-fit line from the linear regression is stored. Students should pay particular attention to the *y*-intercept of the best-fit line. Students should then answer questions 1–3 on pages 1.8 and 1.9. The questions, and their answers in italics, are given below. Discuss the calculated value of the intercept (zero length*)* with students. They should conclude that the relationship is not, in fact, a linear relationship.



Q1. What does your calculated linear regression model predict for the period of a pendulum with zero length?

A. *0.848 sec*

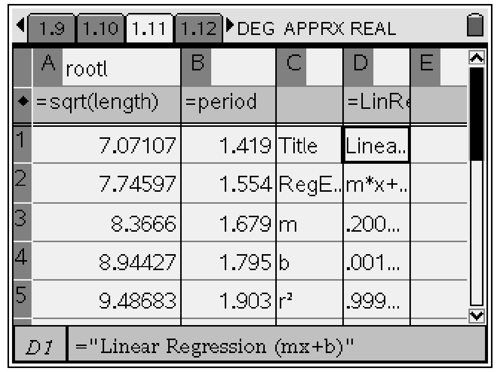
Q2. What should the period of a pendulum of zero length be?

A. *0 sec*

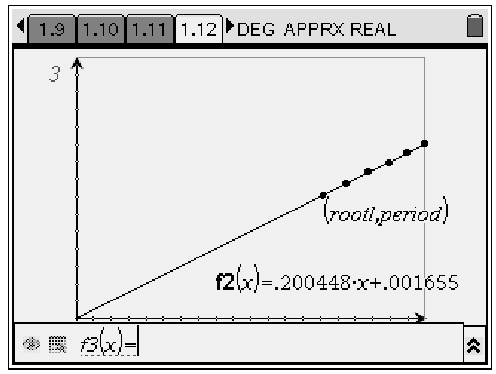
Q3. Based on your linear regression model, is the relationship between period and pendulum length linear? Explain your answer.

A. *The linear regression does not yield the correct period for a zero-length pendulum. Therefore, the relationship between period and pendulum length is probably not linear.*

Part 2 – Building a correct model for the relationship



Step 1: Next, students will attempt to find the best-fit equation of the form *period* = $a\sqrt{L}$ to model the data. Students should read page 1.10 and then move on to the *Lists & Spreadsheet* application on page 1.11. This spreadsheet has the square root of the length in Column A and the period in Column B. Students should perform a linear regression with **rootl** for the *X List* and **period** for the *Y List*. Students should store the values in Column C of the *Lists & Spreadsheet* application on page 1.11.



Step 2: Next, students should create a scatter plot of **period** vs. **rootl**. They should also plot the best-fit line for the data. For this plot, students should not resize the window. They should examine the *y*-intercept of the best-fit line and use their observations to help them answer questions 4–6 on pages 1.13 and 1.14.

Q4. What is the slope of the best-fit line relating **period** and **rootl**?

A. *0.2004 sec/cm$^{1/2}$*

Q5. Does this best-fit equation correctly predict the period of a pendulum with zero length?

A. *Yes.*

Q6. The standard equation for period is as follows:

$$p = 2\pi\sqrt{\frac{L}{g}}$$

where *L* is the pendulum length and *g* = 9.807 m/s^2. Do your results confirm this relationship? Show your work.

A. *Setting the best-fit expression for period equal to the ideal expression for period yields the following:*

$$2\pi\sqrt{\frac{L}{g}} = 0.2004\sqrt{L} + 0.002$$

$$2\pi\sqrt{\frac{L}{980.7}} = 0.2004\sqrt{L} + 0.002$$

$$0.2006\sqrt{L} = 0.2004\sqrt{L} + 0.002$$

The two models agree well. Note that some students will forget to use the correct units; remind them that the best-fit equation uses centimeters for length, but the value of g *given in the problem uses meters.*

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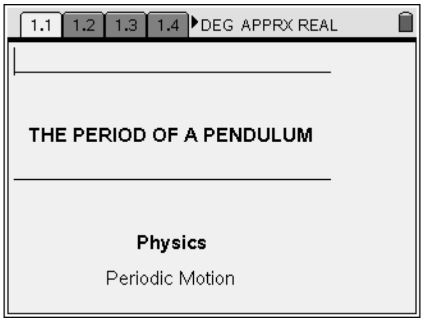
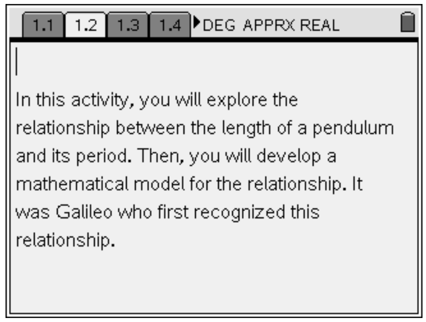
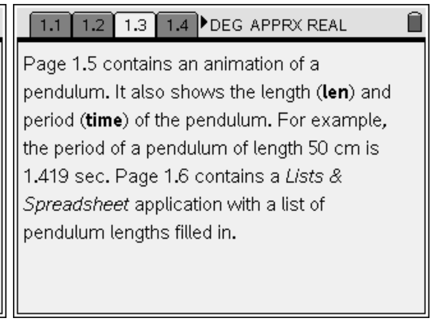
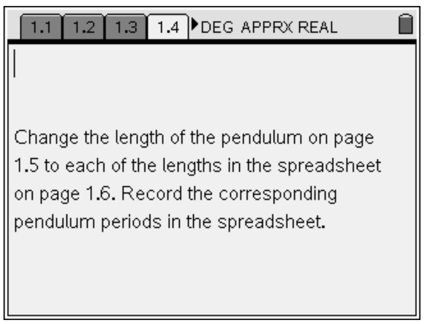
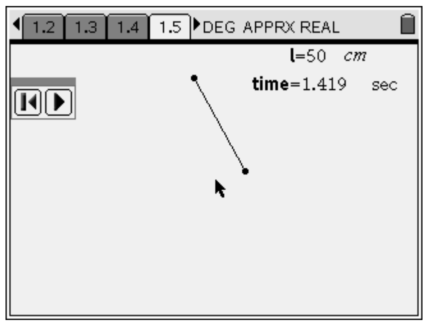
(Student)TI-Nspire File: *PhyAct06_pendulum_EN.tns*

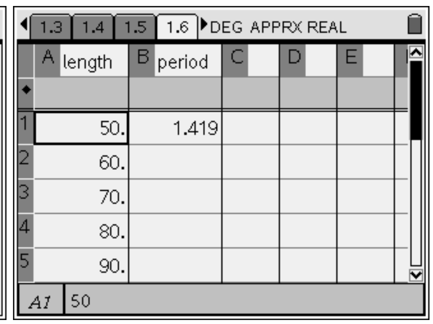


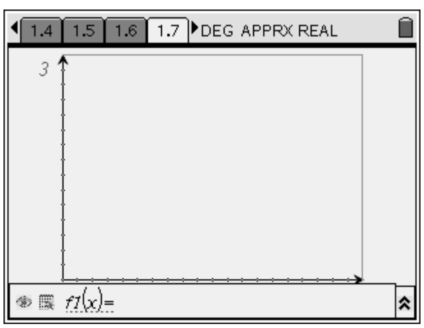


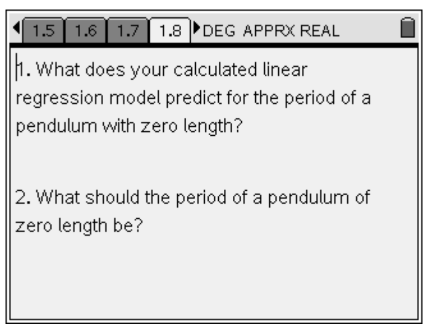


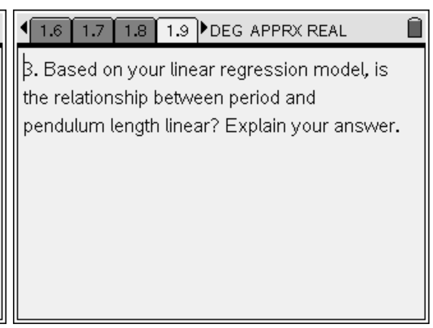


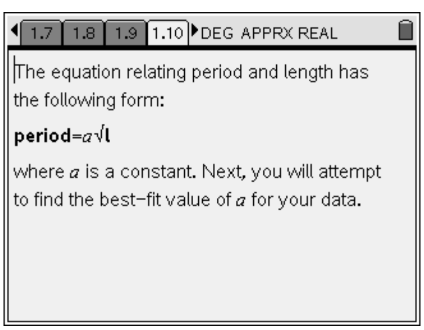


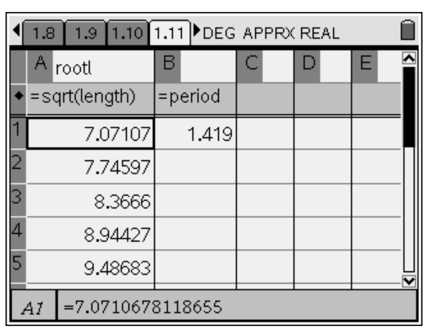


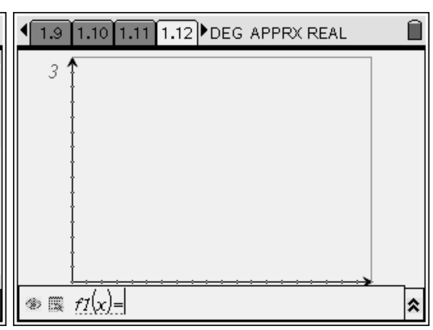












1.10 1.11 1.12 1.13 DEG APPRX REAL

4. What is the slope of the best-fit line relating **period** and **rootl**?

5. Does this best-fit equation correctly predict the period of a pendulum with zero length?

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6. The standard equation for period is as follows:

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where L is the pendulum length and g=9.807 m/s^2. Do your results confirm this relationship? Show your work.